Math 214 – Introductory Statistics **Summer 2008**
6-12-08 Class Notes

Section 4.4
4.4: 3, 8, 13, 16, 19, 26, 30, 31, 34

Conditional Probability

Example 1: (a) Given an ordinary 6-sided die, roll it twice. What is the probability that you will get two 5's?

(b) Given a standard deck of cards, draw one card, don't replace it, and then draw a second card. What is the probability that you will get two kings?

What is different about these two experiments? In each case, we understand each event (call them A and B respectively) separately, but now we are trying to find the probability of A and B. In the last section, we saw that there were two formulas for $P(A \text{ or } B)$ depending on whether the events were mutually exclusive or not. There are also two formulas for $P(A \text{ and } B)$, but this time what matters is whether the events are *independent* or *dependent*.

Definition: Two events A and B are ***independent*** if the fact that A occurs does not affect the probability of B occurring. If this is not the case (i.e. A occurring affects the probability of B occurring), then the events are ***dependent***.

In our example, which is an example of independent events and which is an example of dependent events?

Multiplication Rule For Probabilities

1. If A and B are independent events, then
$$P(A \text{ and } B) = P(A) \cdot P(B).$$

2. If A and B are dependent events, then
$$P(A \text{ and } B) = P(A) \cdot P(B \mid A).$$
(where $P(B \mid A)$ is the probability of B given that A has occurred)

We can use these formulas to solve Example 1.

(a) $P(\text{two } 5's) = P(5 \text{ and } 5) = P(5) \cdot P(5) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

(b) $P(\text{two K's}) = P(K \text{ and K}) = P(K) \cdot P(K|\text{K}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$

Example 2: If 25% of US federal prison inmates are not US citizens, find the probability that two randomly selected inmates will not be US citizens.

$$P(\text{two NC}) = P(\text{NC and NC}) = P(\text{NC}) \cdot P(\text{NC}) = .25 \cdot .25 = .0625$$

Example 3: In a scientific study there are 8 guinea pigs, 5 of which are pregnant. If 3 are selected at random (without replacement), find the probability that all three are pregnant.

$$P(\text{three } P) = P(P \text{ and P and P}) = \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{5}{28}$$

Example 4: In a pizza restaurant, 95% of the customers order pizza, and 65% of customers order a pizza and a salad. Find the probability that a customer who orders a pizza will order a salad too.

$$\begin{aligned} P(\text{P and S}) &= P(\text{P}) \cdot P(\text{S} \mid \text{P}) \\ .65 &= .95 \cdot P(\text{S} \mid \text{P}) \\ \frac{.65}{.95} &= P(\text{S} \mid \text{P}) \\ .684 &= P(\text{S} \mid \text{P}) \end{aligned}$$